B.Tech. 2nd Semester F Scheme

Examination, May–2014

MATHEMATICS-II

Paper–Math–102–F

Common for all branches

Time allowed : 3 hours] *[Maximum marks : 100*

Note : ***Question No. 1 is compulsory.*** *Attempt five questions in total.*

1. (a) Define constant vector.

   (b) Find value of curl (grad ϕ).

   (c) Solve $(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0$.

   (d) Solve $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$.

   (e) Find the Laplace transform of $e^{-3t}\cos^2 t$.

   (f) Define Laplace transform of periodic function.

   (g) Solve $\frac{\partial^2 u}{\partial x\,\partial t} = e^{-t}\cos x$.

   (h) Solve $z = px + qy - 2\sqrt{pq}$.

Section–A

2. (a) Calculate the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and (3, 3, –3).

(b) A vector field is given by

$\vec{A} = (x^2 + xy^2)\ \hat{i} + (y^2 + x^2y)\ \hat{j}$. Show that the field is irrotational and find the scalar potential.

3. (a) Find the circulation of $\vec{F}$ round the curve c, where $\vec{F} = e^x \sin y\ \hat{i} + e^x \cos y\ \hat{j}$ and c is the rectangle whose vertices are

$(0, 0), (1, 0), \left(1, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$.

(b) Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.

Section–B

4. (a) Solve $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$.

(b) When a switch is closed in a circuit containing a battery E, a resistance R and an inductance L, the current i builds up at a rate given by $L\frac{di}{dt} + Ri = E$. Find i as a function of t. How long will it be, before the current has reached one half its maximum value if E = 6 volts, R = 100 ohms and L = 0.1 henry ?

5. (a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

(b) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$.

Section–C

6. (a) Find the inverse Laplace transform of

$$\frac{1}{s^3(s^2+a^2)}.$$

(b) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+a}{b}\right)$.

(c) Apply convolution theorem to evaluate

$$L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right].$$

7. (a) Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3\cos 3t - 11\sin 3t$ given that $y(0) = 0$ and $y'(0) = 6$ by Laplace transform.

(b) Solve $\int_0^t \frac{y(u)}{\sqrt{t-u}}\,du = \sqrt{t}$ by Laplace transform.

Section–D

8. (a) Form the partial differential equation by eliminating the arbitrary functions from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

(b) Solve $x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y)$.

(c) Solve $2zx - px^2 - 2qxy + pq = 0$.

9. (a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions

$u(x, 0) = 3 \sin n\pi x$, $u(0, t) = 0$, $u(1, t) = 0$, where $0 < x < 1, t > 0$.

(b) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within the rectangle $0 \le x \le a, 0 \le y \le b$ given that

$u(0, y) = u(a, y) = u(x, b) = 0$ and $u(x, 0) = x(a - x)$.